

Quantum Computing on Multi-atomic Ensembles in Quantum Electrodynamics Cavity

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We propose an effective realization of a complete set of elementary quantum gates in the solid-state quantum computer based on the multi-atomic coherent (MAC-) ensembles in the QED cavity. Here, we use the two-ensemble qubit encoding and swapping-based operations that together provide implementation of any encoded single-qubit operation by three elementary gates and the encoded controlled-NOT operation is performed in a single step. This approach simplifies a physical realization of universal quantum computing and adds the immunity to a number of errors. We also demonstrate that the proposed architecture of quantum computer satisfies DiVincenzo criteria.

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I. INTRODUCTION

During the last two decades different types of quantum computer and its physical implementations have been considered [1–4], where single natural or artificial atoms, ions, molecules etc., are used for encoding of the qubits. Physical implementation of the quantum computing on many qubits remains a main challenge in the first turn because of decoherence problems causing intensive search for the novel experimental approaches. The promising approach is that using natural atoms (ions, molecules, ...) with long coherence time.

Recently, a new physical realization of a quantum computer has been proposed which uses multi-atomic coherent (MAC) ensembles for encoding of single qubits [5, 6]. MAC ensembles provide a huge enhancement of the effective dipole moment that leads to a considerable acceleration of the quantum processing rate. However, excess excited quantum states in the MAC- ensemble should be blocked in order to realize an effective two-level system providing thereby perfect encoding of the qubits. A dipole-dipole interaction is intensively discussed for the blockade of the excess quantum states [7]. However, the dipole blockade mechanism introduces the decoherence due to a strong dependence of the dipole-dipole interaction on a spatial distance between the interacting atoms. Recently, another blockade mechanism based on using a light-shift imbalance in a Raman transition has been proposed [8] though its implementation is complicated because of additional quantum transitions arising with this imbalance. We have also proposed a novel decoherence free blockade mechanism [9–11] based on the collective interactions of the atoms in the QED cavity. Rapid development of physics and technology of the microcavities [12–14] makes this blockade mechanism a quite promising

though not very simple for experimental realization.

By developing this approach, we demonstrate in this paper how logical single and two-qubit gates can be realized naturally in the quantum computer based on the MAC- ensembles in the QED cavity using multi-qubit encoding intensively discussed in [15–25]. Here, we explicitly show an encoded universality for some set of these gates by using the 2-qubit encoding. This encoding allows solving two major problems of solid-state quantum computing. First of all, it eliminates the problems with implementation of single-qubit operations. Additionally, the encoding forms a decoherence-free subspace (DFS), which allows preventing a number of computational errors. Moreover, this approach opens the ability of performing the controlled-NOT operation in a single step. This operation is achieved by using an additional non-linear frequency shift of the atomic transitions due to Heisenberg-type atom-atom interactions arising in the QED cavity. Finally we show that the proposed implementation of the quantum gates set can be applied for the construction of quantum computer satisfying DiVincenzo criteria [26].

II. SWAPPING GATES

We consider the realization of swapping gates controlled by a photon qubit. We can launch signal photon qubits (denoted by E_{in} and E_{out}) in the QED cavity and take out the qubits in the free space through a semi-transparent mirror Fig. (1). Many photon qubits can be efficiently stored in quantum memory node also situated in the QED-cavity [27, 28] and then can be transferred to the processing nodes on demand [11] for implementation of single and two-qubit gates. For realization of controlled swapping gate, we consider a logical qubit encoded on two MAC ensembles used as processor nodes situated in two distant positions inside the common QED-cavity. First, signal photon is transferred to one of the processor

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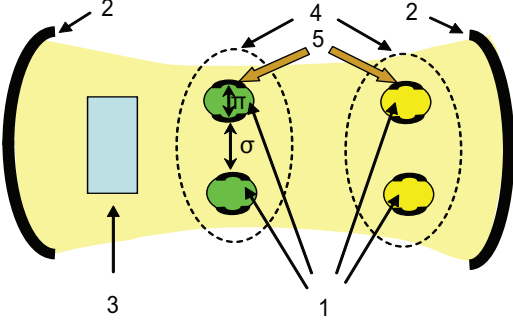


FIG. 1: Scheme of a quantum computer based on multi-atomic ensembles (1) in single mode QED cavity with mirrors (2). The QED-cavity is coupled to external flying photon qubits. Quantum memory (3) is used for storage of the photon qubits. The qubits are transferred to two pairs of processing nodes (green and yellow pairs) for realization of single and two qubit gates. Each pair of nodes ((4) in dashed oval) is used for encoding of a single qubit. Arrows (5) show microcavities, π and σ are the microcavity and cavity modes.

nodes from the quantum memory or another logical qubit and the atomic frequency in the processor nodes is tuned out of the cavity- mode frequency. With that, mirrors are chosen to be dumb at new frequency of processing nodes. The control photon is released from the node with control qubit and is introduced into microcavity with the processing node via the interaction with one MAC ensemble of logical qubit. Here, initially the control photon transfers to the MAC ensemble due to the resonant interaction with additional atomic resonant transitions when the frequency of control qubit node being detuned from the resonant interaction. Then the control photon is released from the MAC ensemble at frequency of the microcavity mode after tuning the frequency of additional atomic transition to that of the microcavity. Afterwards, the frequency of the additional atomic transition is detuned from the frequency of the microcavity mode when control photon is already released. The released control photon cannot be absorbed by MAC ensemble or escape from the microcavity.

Let's introduce the following basis states of the atoms in two nodes 1 and 2: $|\psi\rangle_1 = |1\rangle_1 |0\rangle_2$ and $|\psi\rangle_2 = |0\rangle_1 |1\rangle_2$, where in the case of sample that is small comparing with wavelength of the cavity mode we use collective states $|0\rangle_m = |0_1, 0_2, \dots, 0_{N_m}\rangle$, $|1\rangle_m = \frac{1}{\sqrt{N_m}} \sum_j |0_1\rangle |0_2\rangle \dots |1_j\rangle \dots |0_{N_m}\rangle$ for m-th node (N_m is a number of atoms in m-th node) By taking into account a presence of vacuum and single photon states in the field modes π_1 and π_2 of 1-st and 2-nd microcavities, we write

the total wave function as follows

$$|\Psi\rangle = \sum_{n_{\pi_1}=0}^1 \sum_{n_{\pi_2}=0}^1 |n_{\pi_1}\rangle |n_{\pi_2}\rangle \{c_1(n_{\pi_1}, n_{\pi_2}) |\psi\rangle_1 + c_2(n_{\pi_1}, n_{\pi_2}) |\psi\rangle_2\}, \quad (1)$$

where n_{π_1} and n_{π_2} are the numbers of photons in the microcavity modes.

Hamiltonian of system is written as $H = H_0 + H_1$ where $H_0 = H_a + H_r$ is the main Hamiltonian and $H_1 = H_{r-a}$ is perturbation Hamiltonian. Here, $H_a = H_{a_1} + H_{a_2}$ is a Hamiltonian of the atoms in nodes 1 and 2 and $H_r = H_\sigma + H_\pi$ is a Hamiltonian of photons in mode σ of common QED cavity, so that $H_\pi = H_{\pi_1} + H_{\pi_2}$. With that $H_{a_1} = \hbar\omega_0 \sum_{j_1} S_{j_1}^z$ and $H_{a_2} = \hbar\omega_0 \sum_{j_2} S_{j_2}^z$, where ω_0 is the working frequency of atoms, $S_{j_1}^z$ and $S_{j_2}^z$ are operators of effective spin $1/2$ for atoms in nodes 1 and 2; $H_\sigma = \hbar\omega_{k_\sigma} a_{k_\sigma}^\dagger a_{k_\sigma}$, is a Hamiltonian of the common QED-cavity field mode, $H_{\pi_1} = \hbar\omega_{k_{\pi_1}} a_{k_{\pi_1}}^\dagger a_{k_{\pi_1}}$ and $H_{\pi_2} = \hbar\omega_{k_{\pi_2}} a_{k_{\pi_2}}^\dagger a_{k_{\pi_2}}$ are the Hamiltonians of the microcavity field modes, where ω_{k_σ} and $\omega_{k_{\pi_1}}, \omega_{k_{\pi_2}}$ are the frequencies of photons with wave vectors \vec{k}_σ and $\vec{k}_{\pi_1}, \vec{k}_{\pi_2}$ of modes σ and π_1, π_2 , $a_{k_\sigma}^\dagger, a_{k_{\pi_1}}^\dagger, a_{k_{\pi_2}}^\dagger$ and $a_{k_\sigma}, a_{k_{\pi_1}}, a_{k_{\pi_2}}$ are creation and annihilation operators of photons in corresponding modes.

For the interaction of photons with two atomic nodes $H_{r-a} = H_{r-a}^{(1)} + H_{r-a}^{(2)}$, we have the following expressions:

$$H_{r-a}^{(\alpha)} = \hbar \sum_{j_\alpha}^{N_\alpha} \left(g_{k_\sigma}^{(\alpha)} \exp\{i\vec{k}_\sigma \vec{r}_{j_\alpha}\} S_{j_\alpha}^+ a_{k_\sigma} + H.C. \right), \quad (2)$$

where $g_{k_\sigma}^{(\alpha)}$ is a constant of photon atom interaction, $S_{j_2}^+$ is a raising operator for effective spin $1/2$ in two-level model, \vec{r}_{j_α} are radius-vectors of atoms j_α in nodes $\alpha = 1, 2$.

We are interested in nonresonant interaction between MAC ensembles and the field modes. Here, one can use unitary transformation of Hamiltonian $H_s = e^{-s} H e^s$ leading in the second perturbation order to:

$$H_s = H_0 + \frac{1}{2} [H_1, s], \quad (3)$$

when the relation $H_1 + [H_0, s] = 0$ holds. Using this relation, we find $s = \sum_{p=1}^4 s_p$, where

$$s_n = \hbar \sum_{j_n}^{N_n} \left(\alpha_n g_{k_\sigma}^{(n)} \exp\{i\vec{k}_\sigma \vec{r}_{j_n}\} S_{j_n}^+ a_{k_\sigma} - H.C. \right), \quad (4)$$

$$s_{n+2} = \hbar \sum_{j_n}^{N_n} \left(\beta_n g_{k_{\pi_n}}^{(n)} \exp\{i\vec{k}_{\pi_n} \vec{r}_{j_n}\} S_{j_n}^+ a_{k_{\pi_n}} - H.C. \right), \quad (5)$$

where $n = 1, 2$; $\hbar\alpha_n = 1/(\omega_{k_\sigma} - \omega_0) = -1/\Delta_\sigma$, $\hbar\beta_n = 1/(\omega_{k_{\pi_n}} - \omega_0) = -1/\Delta_{\pi_n}$.

Substituting expressions (4) and (5) into (3), we get

$$\begin{aligned}
H_s = & \hbar\omega_{k_\sigma} a_{k_\sigma}^+ a_{k_\sigma} + \sum_{m=1,2} \hbar\omega_{k_{\pi_m}} a_{k_{\pi_m}}^+ a_{k_{\pi_m}} + \\
& \hbar\omega_1 \sum_{j_1=1}^{N_1} S_{j_1}^z + \hbar\omega_2 \sum_{j_2=1}^{N_2} S_{j_2}^z + \\
& 2\hbar \sum_{m=1,2} \left\{ \sum_{j_m} \left[\frac{|g_{k_\sigma}^{(m)}|^2}{\Delta_m^{(\sigma)}} a_{k_\sigma}^+ a_{k_\sigma} + \frac{|g_{k_{\pi_m}}^{(m)}|^2}{\Delta_m^{(\pi_m)}} a_{k_{\pi_m}}^+ a_{k_{\pi_m}} \right] S_{j_m}^z + \right. \\
& \left. \sum_{i_m, j_m} \frac{|g_{k_\sigma}^{(m)}|^2}{2\Delta_m^{(\sigma)}} e^{i\vec{k}_\sigma \vec{r}_{i_m j_m}} + \frac{|g_{k_{\pi_m}}^{(m)}|^2}{2\Delta_m^{(\pi_m)}} e^{i\vec{k}_{\pi_m} \vec{r}_{i_m j_m}} S_{i_m}^+ S_{j_m}^- \right\} + \\
& \frac{\hbar}{2} \left(\frac{1}{\Delta_1^{(\sigma)}} + \frac{1}{\Delta_2^{(\sigma)}} \right) \sum_{j_1 j_2} \left\{ g_{k_\sigma}^{(1)} g_{k_\sigma}^{(2)*} e^{i\vec{k}_\sigma \vec{r}_{j_1 j_2}} S_{j_1}^+ S_{j_2}^- + h.c. \right\} + \\
& \frac{\hbar}{2} \sum_{m=1,2} \sum_{i_m} \left(\frac{1}{\Delta_m^{(\sigma)}} + \frac{1}{\Delta_m^{(\pi_m)}} \right) \\
& \left\{ g_{k_\sigma}^{(m)} g_{k_{\pi_m}}^{(m)*} e^{i(\vec{k}_\sigma - \vec{k}_{\pi_m}) \vec{r}_{i_m}} a_{k_\sigma} a_{k_{\pi_m}}^+ S_{i_m}^z + h.c. \right\}. \quad (6)
\end{aligned}$$

The first four terms in the Hamiltonian (6) are unperturbed energy of photons and atoms, the fifth and the sixth terms are atomic energy shifts due to their interaction with photons, the seventh and the eighth terms are intra-node interactions between atoms via virtual photons, the ninth term is inter-node interaction between atoms via virtual photons and the tenth term is inter-mode coupling. Hamiltonian (6) describes non-resonant interaction of atoms with the field modes which conserves the initial photon numbers of the microcavity and common QED-cavity modes at $\Delta_m^{(\sigma)} = -\Delta_m^{(\pi_m)}$ when the last term vanishes due to the destructive interference between modes.

By assuming a possibility of single photon excitation only in π_1 - microcavity mode, we analyze the resonant interaction of two MAC-ensembles via exchange of virtual photons of the common QED cavity mode. Using the wave function (1) for the case $n_{\pi_2} = 0$, we obtain the following Schrodinger equation:

$$\begin{aligned}
\frac{d|\Psi(t)\rangle}{dt} = & -\frac{i}{\hbar} H |\Psi(t)\rangle = \\
& i \sum_{n_{\pi_1}=0}^1 \left\{ [\varpi_1(n_{\pi_1}) c_1(n_{\pi_1}, 0) - \sqrt{N_1 N_2} \Omega_\sigma c_2(n_{\pi_1}, 0)] |\psi_1\rangle + \right. \\
& \left. [\varpi_2(n_{\pi_1}) c_2(n_{\pi_1}, 0) - \sqrt{N_1 N_2} \Omega_\sigma^* c_1(n_{\pi_1}, 0)] |\psi_2\rangle \right\} |n_{\pi_1}\rangle |0\rangle, \quad (7)
\end{aligned}$$

where

$$\begin{aligned}
\varpi_1(n_{\pi_1}) = & \left(\frac{N_1}{2} - 1 \right) \left(\omega_1 + 2n_{\pi_1} \Omega_1^{(\pi_1)} \right) + \\
& \frac{N_2}{2} \omega_2 - N_1 \left(\Omega_1^{(\sigma)} + \Omega_1^{(\pi)} \right), \quad (8)
\end{aligned}$$

$$\begin{aligned}
\varpi_2(n_{\pi_1}) = & \frac{N_1}{2} \left(\omega_1 + 2n_{\pi_1} \Omega_1^{(\pi_1)} \right) + \\
& \left(\frac{N_2}{2} - 1 \right) \omega_2 - N_2 \left(\Omega_2^{(\sigma)} + \Omega_2^{(\pi_2)} \right), \quad (9)
\end{aligned}$$

where $\Omega_\sigma = \frac{g_{k_\sigma}^{(1)} g_{k_\sigma}^{(2)*}}{2\hbar} \left(\frac{1}{\Delta_1^{(\sigma)}} + \frac{1}{\Delta_2^{(\sigma)}} \right)$, $\Omega_1^{(\sigma, \pi_1)} = \frac{|g_{k_\sigma, \pi_1}^{(1)}|^2}{\hbar \Delta_1^{(\sigma, \pi_1)}}$ and $\Omega_2^{(\sigma, \pi_2)} = \frac{|g_{k_\sigma, \pi_2}^{(2)}|^2}{\hbar \Delta_2^{(\sigma, \pi_2)}}$.

From (7) we find the following equation for coefficients $c_1(n) \equiv c_1(n_{\pi_1}, 0)$, $c_2(n) \equiv c_2(n_{\pi_1}, 0)$ (where $n \equiv n_{\pi_1}$)

$$\frac{d}{dt} c_1(n) = i\varpi_1^{(n)} c_1(n) - i\sqrt{N_1 N_2} \Omega_\sigma c_2(n), \quad (10)$$

$$\frac{d}{dt} c_2(n) = i\varpi_2^{(n)} c_2(n) - i\sqrt{N_1 N_2} \Omega_\sigma^* c_1(n), \quad (11)$$

Assuming the initial state with the excited first MAC-node $c_1(n) = 1$ and $c_2(n) = 0$ we find the solution

$$c_1(n) = e^{-i\varpi(n)t} \{ \cos(\kappa t) + i \frac{\Delta(n)}{2\kappa(n)} \sin(\kappa(n)t) \}, \quad (12)$$

$$c_2(n) = -ie^{-i\varpi(n)t} \frac{S}{\kappa(n)} \sin(\kappa(n)t), \quad (13)$$

where $\Delta(n)/2 = n\Omega_1^{(\pi)}$, $\kappa(n) = \sqrt{n^2 \left(\Omega_1^{(\pi)} \right)^2 + |S|^2}$, $S = \sqrt{N_1 N_2} \Omega_\sigma$, $\varpi(n) = \frac{1}{2} \left(\varpi_1^{(n)} + \varpi_2^{(n)} \right)$ and for convenience we have used the atomic parameters satisfying the condition $\omega_2 - \omega_1 + N_2 \Omega_2^{(\sigma)} - N_1 \Omega_1 = 0$ (where $\Omega_1 \equiv \Omega_1^{(\sigma)} + \Omega_1^{(\pi)}$).

It can be seen in Eqs. (12), (13) that we have a strong blockade of excitation transfer between the nodes at the presence of control photon ($n = 1$): $c_2(n = 1) \cong 0$ and $c_1(n = 1) \cong \exp\{-i[\varpi(n) + \Omega_1^{(\pi)}]t\}$ for sufficiently high quality factor of the microcavity where $\Omega_1^{(\pi)} \gg S$. Thus, there is no swapping between the states $|\psi_1\rangle$ and $|\psi_2\rangle$ of logical qubit in the presence of control photon. While in the absence of control photon ($n = 0$), we have the following oscillating solutions:

$$c_2 = -ie^{-i\varpi(n)t} \sin(St), \quad (14)$$

$$c_1 = e^{-i\varpi(n)t} \cos(St), \quad (15)$$

demonstrating periodical transfer of excitation between $|\psi_1\rangle$ and $|\psi_2\rangle$ that is a realization of Controlled-SWAP operation.

It is worth noting that the state $|\psi_1\rangle$ is transformed into the state $|\psi_2\rangle$ at short time intervals $t_{C-iSWAP} = \pi/(2N\Omega_S)$. By using of interaction constants $g_\pi \sim 10^{10} \text{ Hz}$ and $g_\sigma \sim 10^6 \text{ Hz}$, we get that the number of atoms must be limited by sufficiently large quantity $N \leq 10^4$, the time of energy transfer is $t_{C-iSWAP} \sim 10^{-8} \text{ sec}$

respectively. So, we come to the realization of nano-dimensional swapping gates controlled by the photon state. We call this gate Controlled-iSWAP(θ) gate because we have here the controlled rotation of qubit on any desired angle $\theta = St$.

Another interesting case of the Controlled-iSWAP gate can be realized for special quantum dynamics of Eqs. (12) and (13). Here, by using evolution time $\tilde{t} = \pi/(2S)$ with interaction parameter $|\Omega_1^{(\pi)}| = \sqrt{3}|S|$ one can provide a perfect blockade of iSWAP operation due to $\kappa_{(1)}\tilde{t} = \pi$. Eventual truth table for the iSWAP gate controlled by the two states of photon ($|0\rangle$ and $|1\rangle$) is the following

	$ \psi\rangle_1 0\rangle$	$ \psi\rangle_2 0\rangle$	$ \psi\rangle_1 1\rangle$	$ \psi\rangle_2 1\rangle$
$ \psi\rangle_1 0\rangle$	0	$-i$	0	0
$ \psi\rangle_2 0\rangle$	$-i$	0	0	0
$ \psi\rangle_1 1\rangle$	0	0	$e^{-i(\varpi(1)-\varpi(0))\tilde{t}}$	0
$ \psi\rangle_2 1\rangle$	0	0	0	$e^{-i(\varpi(1)-\varpi(0))\tilde{t}}$

Table 1. Truth table for the Controlled-iSWAP gate. This case imposes more mild relation between the quality factors of the microcavities and common cavity. We note that it is possible also to realize the Controlled-iSWAP gate where presence of a single photon will equalize the initially different node frequencies leading thereby to the swapping process in an opposite manner to that in Table 1.

Summarizing, we note that realization of the Controlled-iSWAP gate provides together with the iSWAP gate a Controlled-NOT gate by using logical qubit encoding by two physical qubits. Below we discuss the main issues related to a realization of complete set of gates for universal quantum computations.

III. UNIVERSAL QUANTUM COMPUTING

The principal scheme of a quantum computer with multiatomic ensembles in the cavity is shown in Fig.1. Quantum memory is used for storage of many photon qubits. The qubits are transferred to two pairs of processing nodes (blue and green pairs) for realization of single- and two qubit gates. Each pair of nodes (in dashed oval) is used for encoding of a single qubit.

According to DiVincenzo any quantum computer should have: 1. Scalability, 2. Possibility of initialization, 3. Possibility of read-out, 4. Limited decoherence, 5. Availability of a universal set of quantum gates. Let's consider main DiVincenzo criteria [26] for quantum computer in our architecture.

1. Scalability. Our construction is scalable since the multi-qubit quantum memory and the processing nodes are situated in common QED cavity [11] and all nodes can be connected by swap process providing the quantum operations over the large number of qubits stored in the quantum memory.

2. Initialization. Arbitrary initial quantum state of the system can be reliably initialized and downloaded to the quantum memory node and qubits can be transferred from quantum memory node to any processing node on demand [11].

3. Read-out. A qubit state can be efficiently read out in a single-shot fashion by detecting photon echo signals from the quantum memory incorporated in the system [27].

4. Decoherence. The system operates in decoherence free subspace and all decoherence is connected with small inclinations from the model presented here. Main sources of the decoherence are atomic phase relaxation (Γ) and cavity losses (γ). In this case we can generalize our result by using Walls-Milburn input-out formalism [29] for evaluation of the fidelity for iSWAP operation $F = |\langle\psi_{out}(\Gamma, \gamma) | \psi_{out}^{ideal}\rangle|^2 = \exp\{-2\Gamma t_{iSWAP} - \pi\gamma/2\Delta\} \cosh^2(\pi\gamma/4\Delta)$ (where $t_{iSWAP} = \pi/(2N\Omega_\sigma)$). Thus realization of fault-tolerant quantum computing [30] requires the following values of the relaxation parameters $2\Gamma t_{iSWAP} + \pi\gamma/2\Delta \leq 10^{-4}$ that makes preferable using the spin transitions with low decay constants and weak quantum noise Γ [31] in the QED cavities with high quality factor [32]. Similar requirements occur for iSWAP(θ), Controlled-iSWAP(θ) and for other quantum gates in our architecture.

5. Universal set of quantum gates. The operation of a logical single qubit (iSWAP) gate and two qubit gate (Control-iSWAP) was considered in the previous chapter. Let's discuss the following property explicitly showing that in encoding of qubits used here the two and three qubit operations available in our physical model are sufficient to implement the standard set, thus proving their encoded universality.

Property 2.1. The set of quantum gates {Controlled-iSWAP; iSWAP(θ); PHASE(ϕ)} is universal for the Hilbert subspace spanned by encoded states $|0_L\rangle = |01\rangle$, $|1_L\rangle = |10\rangle$ (where PHASE(ϕ)- gate is realized by adjustable shifting of the atomic frequency $\Delta\omega$ in one of the two physical qubits during the fixed time interval $t' = \phi/\Delta\omega$).

Proof. First of all we show the effect of our elementary quantum gates when acting on pairs of nodes in basis states $|01\rangle$, $|10\rangle$ and their linear combinations.

For instance, the iSWAP operation turns $|01\rangle$ into $|10\rangle$ and backwards, thus acting on a pair like the gate X (the NOT gate). In the similar manner Controlled-iSWAP actually implements the logical CNOT gate.

More formally:

$$\text{CNOT}_L (\alpha_1|0_L\rangle + \alpha_2|0_L\rangle + \alpha_3|1_L\rangle + \alpha_4|1_L\rangle =$$

$$\text{C(SWAP)} (\alpha_1|01\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|10\rangle =$$

$$\alpha_1|01\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|10\rangle =$$

$$\alpha_1|0_L\rangle + \alpha_2|0_L\rangle + \alpha_3|1_L\rangle + \alpha_4|1_L\rangle =$$

If we look at the matrix for the generalized iSWAP(θ)

gate, it's middle part (responsible for transforming $|01\rangle$ and $|10\rangle$ basis states) is actually a rotation by the angle $-\theta$ about the x axis of the Bloch sphere, i.e. $i\text{SWAP}(\theta)$ corresponds to the following operator:

$$i\text{SWAP}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \frac{\theta}{2} & i \sin \frac{\theta}{2} & 0 \\ 0 & i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow$$

$$R_x(\theta) = \begin{pmatrix} \cos \frac{\theta}{2} & i \sin \frac{\theta}{2} \\ i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}.$$

Similarly, $\text{PHASE}(\theta)$ turns our composite qubit around the axis z (up to the phase factor of $e^{i\varphi/2}$):

$$\text{PHASE}(\theta) = e^{i\varphi/2} \begin{pmatrix} e^{-i\varphi/2} & 0 & 0 & 0 \\ 0 & e^{-i\theta/2} & 0 & 0 \\ 0 & 0 & e^{i\theta/2} & 0 \\ 0 & 0 & 0 & e^{i\varphi/2} \end{pmatrix} \rightarrow$$

$$R_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & \\ & e^{i\theta/2} \end{pmatrix}. \quad (16)$$

Since an arbitrary rotation of a single qubit (and thus any single qubit gate) can be decomposed into the product of three rotations about orthogonal axes (say, R_x and R_z), our basis allows to avoid using operations of single processing nodes and thus blockage. All of the single qubit gates are performed by the means two node operations.

For instance, the Hadamard transform can be implemented as follows:

$$H = e^{i\pi/2} R_z\left(\frac{\pi}{2}\right) R_x\left(\frac{\pi}{2}\right) R_z\left(\frac{\pi}{2}\right). \quad (17)$$

The other two single qubit gates S and T from the standard set up to the phase factor are rotations about z axis:

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = e^{i\pi/4} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix} = e^{i\pi/4} R_z\left(\frac{\pi}{2}\right), \quad (18)$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} = e^{i\pi/8} \begin{pmatrix} e^{-i\pi/8} & 0 \\ 0 & e^{i\pi/8} \end{pmatrix} = e^{i\pi/8} R_z\left(\frac{\pi}{4}\right). \quad (19)$$

Therefore, the set of gates $\{\text{Controlled-SWAP}; i\text{SWAP}(\theta); \text{PHASE}(\phi)\}$ allows to implement the standard set of quantum gates, which proves it's encoded universality.

Note, that the universality of the proposed set of elementary gates rely on the presence of "continuous" operations $i\text{SWAP}(\theta)$ and $\text{PHASE}(\theta)$. This fact requires

a higher precision of the hardware, but excludes the approximation algorithms for implementing arbitrary single qubit operations using the standard set of CNOT, H, S, and T. Conversely, we may restrict ourselves with using only $i\text{SWAP}(\pi/2)$, $\text{PHASE}(\pi/2)$ and $\text{PHASE}(\pi/4)$ (which is enough for implementing gates H, S, and T) and exploit standard approximation schemes for arbitrary single qubit gates.

It is also well-known that the Controlled-SWAP (Fredkin) gate is universal for classical computations, since it can be used to perform logical NOT, AND, and FANOUT operations. Therefore, any classical computations can be also performed by a quantum computer of the proposed architecture.

IV. CONCLUSION

Summarizing, we have proposed an approach for constructing encoded universal quantum computations on the multi-atomic coherent ensembles based on swapping operations in QED cavity. The main ideas of the proposed quantum computing scheme are encoding of logical qubits by two physical qubits and using microcavities for implementing Controlled- $i\text{SWAP}$ operations. Scalability is provided by using of multi-qubit quantum memory situated in one of the multi-atomic node. It allows to implement explicitly any encoded single-qubit gate by 3 elementary operations and to perform encoded controlled-NOT gate by a single Controlled-SWAP operation on pairs of atomic ensembles. The physical implementation of the basic gates is sufficiently robust and provides fast single qubit operations based on multi-atomic ensembles. Detailed analysis of these issues requires further investigation of physical limitations determined by the experimental parameters of real physical systems.

The proposed approach considerably simplifies physical implementation of a quantum computer on multi-atomic ensembles in the QED cavity at the price of doubling the number of qubits for computation. Besides, it permits to avoid the necessity of implementing blockade of excess states in the multi-atomic ensembles. Note also, that using of two atomic ensembles for encoding of a single qubit state will be convenient for the quantum computer interface with the external quantum information carried by using photon polarization qubits since the two polarization components of the photon qubit can be coupled directly with the relevant pair of the atomic ensemble state.

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